University of Tripoli - Faculty of Engineering

Department of Electrical and Electronics Engineering

EE302 Signals and Systems

2nd Mid-Term Exam Solution, Fall 2017, 17 December, 2017

Q1 – a)

$$h(t) = h1(t) - \delta(t-1) * h1(t)$$

$$\delta(t-1) * h1(t) = \int_{-\infty}^{\infty} \delta(\tau - 1) h1(t-\tau) d\tau = h1(t-1) \int_{-\infty}^{\infty} \delta(\tau-1) d\tau = h1(t-1)$$

$$h(t) = h1(t) - h1(t-1) = 5u(t) - 10u(t-1) + 5u(t-2)$$

b) If the input signal is $x(t) = 3\delta(t) + \delta(t-1)$

Then the output signal is given as:

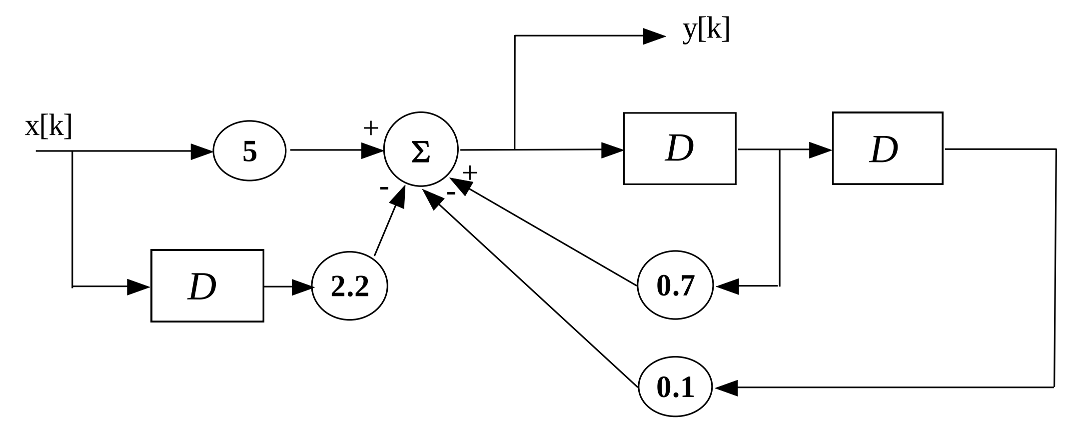
$$y(t) = 3h(t) + h(t-1) = 15u(t) - 25u(t-1) + 5u(t-2) + 5u(t-3)$$

Q2 – a)

$$\lambda_1 = \frac{1}{5}, \lambda_2 = \frac{1}{2} \quad \longrightarrow \quad (\lambda - \lambda_1)(\lambda - \lambda_2) = 0 \quad \longrightarrow \quad \lambda^2 - 0.7\lambda + 0.1 = 0$$

$$h_{k+2} - 0.7h_{k+1} + 0.1h_k = 0$$

as $\quad h_0 = 5, h_1 = 2.2 \quad \longrightarrow \quad y_k - 0.7y_{k-1} + 0.1y_{k-2} = 5x_k - 2.2x_{k-1}$



b) From the given impulse response

$$h_k = 4\left(\frac{1}{5}\right)^k + \left(\frac{1}{2}\right)^k \qquad k \geq 0,$$

Thus output of the input signal $\quad x_k = 2\delta(k-4)$ is given as:

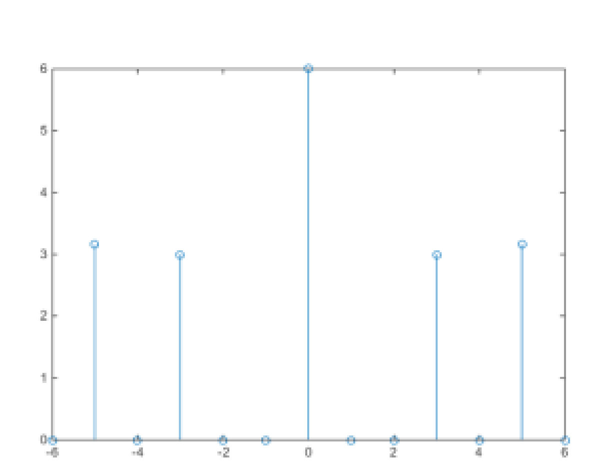
$$y_k = \begin{cases} 0 & k < 4 \\ 8\left(\frac{1}{5}\right)^{k-4} + 2\left(\frac{1}{2}\right)^{k-4} & k \geq 4 \end{cases}$$

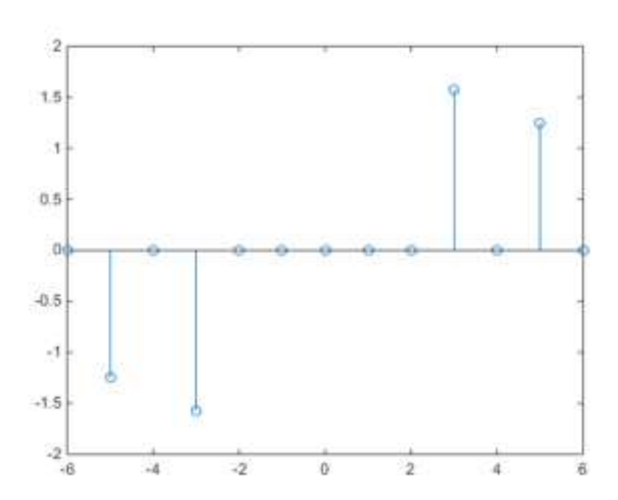
[5] **Q3** –

$$x(t) = (1 - j3)e^{-j5t} - j3e^{-j3t} + 6 + j3e^{j3t} + (1 + j3)e^{j5t}$$

Exponential Fourier series		Compact trigonometric Fourier series		Trigonometric Fourier series	
$D_0=6$	$D_0=6$	$C_0=6$	$\theta_0=0$	$a_0=6$	
$D_1= 0$	$\|D_1\|=\|D_{-1}\|=0$	$C_1=0$	$\theta_1=\theta_{-1}=0$	$a_1=0$	$b_1=0$
$D_2= 0$	$\|D_2\|=\|D_{-2}\|=0$	$C_2=0$	$\theta_2=\theta_{-2}=0$	$a_2=0$	$b_2=0$
$D_3= 3j$	$\|D_3\|=\|D_{-3}\|=3$	$C_3=6$	$\theta_3=-\theta_{-3}=\pi/2$	$a_3=0$	$b_3= -6$
$D_4= 0$	$\|D_4\|=\|D_{-4}\|=0$	$C_4=0$	$\theta_4=\theta_{-4}=0$	$a_4=0$	$b_4=0$
$D_5= 1+3j$	$\|D_5\|=\|D_{-5}\|=3.16$	$C_5=6$	$\theta_5=-\theta_{-5}=6.325$	$a_5=2$	$b_5= -6$

a)





b) The compact trigonometric Fourier series

$C_0=D_0=6$ $\quad$ $C_n=2|D_n|$

$$x(t) = 6 + 6\cos(3t + \pi/2) + 2\sqrt{10}\cos(5t + 1.25)$$

The trigonometric Fourier series

$a_0=C_0=D_0=6$ $\quad$ $a_n-jb_n=2D_n$ $\quad$ $a_n+jb_n=2D_{-n}$

$$x(t) = 6 - 6\sin(3t) + 2\cos(5t) - 6\sin(5t)$$